`shuffle.sty`, a shuffle product symbol *

Julian Gilbey (font) and Antoine Lejay (documentation)

Released 2008/10/27

Abstract

This package provides an interface to the symbol of shuffle product which is used in some area of mathematics such as algebra.

1 The shuffle product symbol

Dependencies: this package uses the METAFONT files `shuffle.mf`, `shuffle7.mf` and `shuffle10.mf`. These files should be put into the `font/source/` directory of your `texmf` tree (do not forget to perform `texhash` or an equivalent command).

This package provides the following two symbols

Shuffle product	⧢	`\shuffle`
Complete shuffle product	⧢̄	`\cshuffle`

In a space of formal non-commutative polynomials whose indeterminates are identified with letters (and then the product of indeterminates are identified with words), the shuffle product $u \sqcup\!\sqcup v$ of two words $u = u_1 \cdots u_n$ and $v = v_1 \cdots v_n$ is defined as the sum of the all words it is possible to construct from u and v by preserving the order of all the letters in each of the words. For example

$$\alpha\beta \sqcup\!\sqcup \gamma = \gamma\alpha\beta + \alpha\gamma\beta + \alpha\beta\gamma.$$

The shuffle product is used in combinatorics and algebra (notably for free Lie algebra [2] and Hopf algebra) and has applications for example in the formal resolution of controlled differential equations.

The symbol ⧢ is a standard notation but is does not seem to have already been provided for LaTeX. The METAFONT source of this symbol has been designed by J. Gilbey for his own usage [1], where the notion of complete shuffle product ⧢̄ is also defined.

The symbol ⧢ is also defined as a Unicode symbol at position `29E2` [3] under the name `SHUFFLE PRODUCT`.

*This file describes version Shuffle, last revised 2008/10/27.

References

[1] J. Gilbey. Permutation Groups, a Related Algebra and a Conjecture of Cameron, *Journal of Algebraic Combinatorics*, 19 (2004) 25–45.

[2] C. Reutenauer. *Free Lie algebras*, Oxford University Press, 1993.

[3] The Unicode Consortium. *The Unicode Standard 5.1*, 2007.

2 The font definition files

The content of this file is standard.

```
\ProvidesFile{Ushuffle.fd}%
\DeclareFontFamily{U}{shuffle}{}
\DeclareFontShape{U}{shuffle}{m}{n}{%
<5-8>shuffle7%
<8->shuffle10%
}{}
```

3 The code

The content of this file is standard. It starts by declaring the font as a symbol font.

```
\NeedsTeXFormat{LaTeX2e}
\ProvidesPackage{shuffle}[2008/10/27 Shuffle product symbol]
\DeclareSymbolFont{Shuffle}{U}{shuffle}{m}{n}
```

`\shuffle` `\cshuffle` Two commands are defined access to the symbols defined as binary symbols.

```
\DeclareMathSymbol\shuffle{\mathbin}{Shuffle}{"001}
\DeclareMathSymbol\cshuffle{\mathbin}{Shuffle}{"002}
```